## Coordinate Systems

**1) Cartesian csys**:

$\boldsymbol{v}_{P|F} = \dot{x}\hat{\boldsymbol{i}} + \dot{y}\hat{\boldsymbol{j}} + \dot{z}\hat{\boldsymbol{k}}$

$\boldsymbol{a}_{P|F} = \ddot{x}\hat{\boldsymbol{i}} + \ddot{y}\hat{\boldsymbol{j}} + \ddot{z}\hat{\boldsymbol{k}}$

**2) Cylindrical polar csys**:

$\boldsymbol{v}_{P|F} = \dot{r}\hat{\boldsymbol{e}}_r + r\dot{\phi}\hat{\boldsymbol{e}}_\phi + \dot{z}\hat{\boldsymbol{e}}_z$

$\boldsymbol{a}_{P|F} = \left(\ddot{r} - r\dot{\phi}^2\right)\hat{\boldsymbol{e}}_r + \left(2\dot{r}\dot{\phi} + r\ddot{\phi}\right)\hat{\boldsymbol{e}}_\phi + \ddot{z}\hat{\boldsymbol{e}}_z$

**3) Path csys**:

$\boldsymbol{v}_{P|F} = \dot{s}\hat{\boldsymbol{e}}_t$

$\boldsymbol{a}_{P|F} = \ddot{s}\hat{\boldsymbol{e}}_t + \frac{\dot{s}^2}{\rho}\hat{\boldsymbol{e}}_n$

$\frac{1}{\rho} = \frac{\left|\boldsymbol{v}_{P|F} \times \boldsymbol{a}_{P|F}\right|}{\left|\boldsymbol{v}_{P|F}\right|^3}$

## Velocity and Acceleration Transfer

$\dot{\boldsymbol{A}}_{|F} = \dot{\boldsymbol{A}}_{|m} + \boldsymbol{\omega}_{m|F} \times \boldsymbol{A}$

$\boldsymbol{v}_{P|F} = \boldsymbol{v}_{P|m} + \boldsymbol{v}_{A|F} + \boldsymbol{\omega}_{m|F} \times \vec{r}_{PA}$

$\boldsymbol{a}_{P|F} = \boldsymbol{a}_{P|m} + \boldsymbol{a}_{A|F} + \dot{\boldsymbol{\omega}}_{m|F} \times \vec{r}_{PA} + 2\boldsymbol{\omega}_{m|F} \times \boldsymbol{v}_{P|m} + \boldsymbol{\omega}_{m|F} \times (\boldsymbol{\omega}_{m|F} \times \boldsymbol{r}_{PA})$

## Composition of angular vel. and acc.

$\boldsymbol{\omega}_{3|1} = \boldsymbol{\omega}_{3|2} + \boldsymbol{\omega}_{2|1}$

$\dot{\boldsymbol{\omega}}_{3|1} = \dot{\boldsymbol{\omega}}_{3|2} + \dot{\boldsymbol{\omega}}_{2|1} + \boldsymbol{\omega}_{2|1} \times \boldsymbol{\omega}_{3|2}$

## Forces and Moments

1) Equivalent Force Systems:

$$\boldsymbol{F}_R = \boldsymbol{F}'_R$$
$$\boldsymbol{M}_A = \boldsymbol{M}'_A$$

2) Linear Momentum:

$$\boldsymbol{p}_{|F} = \int_m \boldsymbol{v}_{P|F}\, dm = m\boldsymbol{v}_{C|F}$$

3) Angular Momentum:

$$\boldsymbol{H}_{A|F} = \int_m \boldsymbol{r}_{PA} \times \boldsymbol{v}_{PA|F}\, dm$$

$$\boldsymbol{H}_A = \mathbf{I}^A \boldsymbol{\omega}_{m|F}$$
$$(\boldsymbol{H}_A)_i = I^A_{ij}\omega_{ij}$$

## Axioms

1) Euler's Axioms (point $O$ fixed in '$I$')

$$\frac{d}{dt}\left\{\boldsymbol{p}_{|I}\right\}\Big|_I = \dot{\boldsymbol{p}}_{|I} = \boldsymbol{F}_R$$
$$\frac{d}{dt}\left\{\boldsymbol{H}_{O|I}\right\}\Big|_I = \dot{\boldsymbol{H}}_{O|I} = \boldsymbol{M}_O$$

2) Modified Euler's 2nd axiom

$$\frac{d}{dt}\left\{\boldsymbol{H}_{A|I}\right\}\Big|_I = \dot{\boldsymbol{H}}_{A|I} = \boldsymbol{M}_A - \boldsymbol{r}_{CA} \times m\boldsymbol{a}_{A|I}$$

3) Coulomb friction axiom:

$$\text{Static}: \ |\boldsymbol{f}| \leq \mu_s\,|\boldsymbol{N}|$$
$$\text{Kinetic}: \ |\boldsymbol{f}| = \mu_k\,|\boldsymbol{N}|$$

## Rigid Body Dynamics

1) Inertia Tensor (and matrix components)

$$I^A_{ij} = \int_m \left(r^2\delta_{ij} - x_i x_j\right) dm$$

$$\mathbf{I}^A = \int_m \left\{(\boldsymbol{r}_{PA} \cdot \boldsymbol{r}_{PA})\,\mathbf{I} - \boldsymbol{r}_{PA} \otimes \boldsymbol{r}_{PA}\right\} dm$$

2) Parallel Axes Theorem

$C$ is the COM, and the origin is at $A$

$$I^A_{11} = I^C_{11} + m\left(x^2_{C_2} + x^2_{C_3}\right)$$
$$I^A_{22} = I^C_{22} + m\left(x^2_{C_1} + x^2_{C_3}\right)$$
$$I^A_{33} = I^C_{33} + m\left(x^2_{C_1} + x^2_{C_2}\right)$$
$$I^A_{ij} = I^C_{ij} - m\, x_{C_i} x_{C_j} \quad i \neq j$$

## Moment of Inertia for Common RBs

1) Rectangular Cuboid ($m = \rho whl$)

$$\left[\mathbf{I}^C\right] = m \ \text{diag}\left(\left[\frac{(w^2+h^2)}{12}, \frac{(l^2+h^2)}{12}, \frac{(l^2+w^2)}{12}\right]\right)$$

2) Circular Solid Cylinder ($m = \rho\pi R^2 l$)

$$\left[\mathbf{I}^C\right] = m \ \text{diag}\left(\left[\left(\frac{R^2}{4} + \frac{l^2}{12}\right), \left(\frac{R^2}{4} + \frac{l^2}{12}\right), \frac{R^2}{2}\right]\right)$$

'diag' denotes the diagonal components of $\left[\mathbf{I}^C\right]$